The viscoelastic-like response of a repulsive granular medium during projectile impact and penetration

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ABSTRACT

We study experimentally the penetration of a projectile into a two-dimensional granular bed composed of magnetic repelling grains. The projectile can experience either repulsive, attractive or neutral interaction with the particles generating different dynamics: i) the repulsive intruder compresses the granular bed without contact and experiences rebounds before stopping; ii) the attractive intruder is first accelerated when it approaches the bed, then, some magnets attach to its surface increasing its effective diameter and the drag force acting on the intruder until it reaches the repose; and iii) the neutral projectile penetrates deeper into the granular bed than in the two previous cases because there is no magnetic interaction with the particles. In addition, we developed molecular dynamics simulations able to reproduce the experimental results and used to determine the effect of the magnetic strength and the role of friction between the particles and the container walls. The results show that the medium behaves similar to a viscoelastic micellar fluid on impact, and the projectile dynamics can be modelled by a spring-dashpot model. Our findings can be useful in the design of new magnetic granular dampers.

1. Introduction

The impact and penetration of a solid object into a liquid or into a granular medium has been widely studied due to its relevance in military and aerospace engineering [1,2], vehicle and animal locomotion [3], or for understanding cratering phenomena [4–7]. Surprising similarities are found during the impact of a projectile on liquids or granular surfaces [8], including jet-like splash, cavity formation and collapse, a complex stress response to the penetration, and power-law scalings [9]. Nevertheless, a remarkable difference between impacts in fluids and grains is that an object sinks in a fluid whereas in a typical granular material the object reaches a final depth. This difference is mainly due to the nature of friction and drag forces acting in each scenario.

The dynamics of an object falling through a medium can be derived by introducing an appropriate drag force \( F_d \) into the equation of motion. This force can depend on the object geometry, its velocity \( \dot{z} \), position \( z \), and on the nature and properties of the medium [10]. In a Newtonian fluid, the drag force on a sphere of radius \( r_s \) scales linearly with \( z \) when viscous forces dominate (low Reynolds numbers \( Re \ll 1 \)) and it is given by the Stokes law:

\[
F_d = 6 \pi r_s \eta \dot{z} \quad \text{(Newtonian fluid, viscous regime)}
\]

where \( \eta \) is the viscosity of the fluid. At large Reynolds numbers (\( Re \gg 1 \)), when inertial forces dominate, \( F_d \) scales with the square of the projectile velocity as:

\[
F_d = C_d \pi r_s^2 \dot{z}^2 \quad \text{(Newtonian fluid, inertial regime)}
\]

where \( C_d \) is the hydrodynamic drag coefficient of the object and depends mainly on the projectile geometry. In a Newtonian fluid, a falling object reaches a terminal velocity when its weight is balanced by the drag force.

On the other hand, when a projectile penetrates into a viscoelastic fluid, a non-Newtonian fluid that combines aspects of an elastic solid and a viscous fluid [11,12], the projectile velocity can exhibit oscillations while approaching to its terminal value [9,13]. A common model to describe viscoelasticity is the Maxwell model, in which the elastic and viscous response of the fluid is approximated by a two-element system connected in series: a linear spring element of elastic constant \( k \) and a linear viscous dashpot element with viscous damping coefficient \( \beta \). In the Maxwell model, it is assumed that the spring and the dashpot behave according to Hooke’s and Stoke’s laws, respectively. Therefore, the drag force is given by:

\[
F_d = k \dot{z} + \beta \dot{z} \quad \text{(viscoelastic fluid)}
\]
Depending on the constants \( k \) and \( \beta \), the solution of the equation of motion can be underdamped, reproducing the velocity oscillations observed during the penetration of a projectile in wormlike micellar fluids [9], or overdamped, as in the case of impacts in polyvinyl alcohol solutions, in which case the object velocity decays exponentially [10].

If the viscoelastic fluid is shear thickening, its viscosity increases when the liquid is subjected to a shear stress. In Ref. [14], an added mass model was proposed to describe the shear thickening response of a solution of cornstarch dissolved in water on a low-velocity impacting object. Nevertheless, in Ref. [10], it was shown that for high-velocity impacts the ability of non-Newtonian liquids to stop objects is given by their viscoelasticity rather than the shear thickening.

Concerning granular materials, a widely accepted model to describe the dynamics of an impacting object considers the drag force as the sum of a depth dependent friction term plus an inertial viscous term [15]:

\[
F_d = F(z) + \gamma z^2 \quad \text{(granular material)}
\]

where \( \gamma \) is a viscous drag coefficient, and \( F(z) \) is the friction term often taken to increase linearly with depth \( z \); therefore, the velocity of the projectile decreases from its value at impact and the friction stops the object at a given final depth where the projectile ends up embedded and stationary. However, for low-packing fractions or ultra-light granular materials, \( F(z) \) has an exponential saturation due to the Janssen effect. Under such conditions, a dense object can sink indefinitely through the grains and reaches a terminal velocity, revealing a liquid-like behaviour as in the case of Newtonian fluids [17,18].

In a granular medium, contacts between particles produce energy dissipation by friction and collisions, and they also confer stiffness to the granular structure through contact force chains [19]. How does the dynamics of a projectile penetrating a granular bed change if the inter-particle collisions are completely suppressed? Would the target behave more like a granular assembly, or like a fluid? To address these fundamental questions, we studied experimentally and using molecular dynamics simulations the impact and penetration of a projectile into a two-dimensional granular bed composed of magnetic repelling particles. We found that, due to the impact and penetration of the projectile, marked differences in the local density of the granular bed above and below the projectile are induced. Below the projectile the bed is compressed, decreasing the separation between magnets and increasing their repulsive magnetic energy. This generates a viscoelastic-like response of the granular bed that can make the projectile bounce. Even though contact force chains do not exist in this repulsive medium, the magnetic interaction induces rigidity in the confined granular bed and the projectile stops at a certain depth. Thus, the medium displays some features reminiscent of viscoelastic fluids and other features related to the granular nature of the material.

To explore the above dynamics, repulsive, attractive and neutral interactions between the intruder and the bed were considered. It is known that a magnetic drop impacting a magnetic fluid surface [20] generates a liquid splash remarkably different from the one generated by the impact of a drop into a viscous liquid [21]. Similarly, we found that the dynamics of the grains perturbed by the magnetic intruder differs from the neutral case due to the long-distance magnetic interaction. In the following, we first describe the experimental setup and the algorithm used in the numerical simulations. Then, the dynamics of the three intruders for different impact velocities is compared; the effect of the magnetic strength and friction with the walls is analyzed, and finally, a spring dashpot model is proposed to describe the intruder dynamics.

2. Experimental setup

A sketch of the experimental set-up is presented in Fig. 1a. A vertical Hele-Shaw cell was built with two transparent glass walls of \( 40 \times 150 \text{ cm}^2 \) separated by an acrylic frame of 3.1 mm width. A set of 550 neodymium magnetic discs (each one of thickness \( t = 3.00 \pm 0.02 \text{ mm} \), diameter \( d = 5.00 \pm 0.02 \text{ mm} \), mass \( m = 0.4 \text{ g} \) and surface magnetic field \( B = 5.9 \text{ kG} \) were introduced into the cell with their dipolar magnetic moment pointing out perpendicular to the walls and with the same orientation in order to obtain repulsive grain-grain interactions. The discs are therefore confined by the lateral, frontal and rear walls under the action of gravity, forming a magnetic repelling two-dimensional granular medium. The system was mounted on two horizontal pivots that allow us to vertically invert the cell by rotation to accumulate the grains in the opposite extreme. Then, a magnetic bar is placed on the outside of the front wall at 80 cm from the bottom of the cell. This bar performs the function of a gate that avoids the flow of grains when the cell is rotated to the vertical position (until here, the system is similar to the one used in our previous study about the discharge of magnetic repelling grains from a silo [22,23]). When the bar is removed, the grains fall under gravity and accumulate at the bottom of the cell forming a granular bed. This procedure allows us to obtain reproducible initial conditions for the bed packing fraction, with the grains distributed in an area of \( 40 \text{ cm} \times 32 \text{ cm} = 1280 \text{ cm}^2 \) (i.e. average area density of 0.43 magnets/cm² and mean packing fraction = 0.08).

Once the bed is ready, a projectile is introduced through a removable gate at the top of the cell and held with the magnetic bar at a given height \( h \) measured from the bed surface (determined by the average position of the upper first line of magnets). The projectile is dropped from \( h = 10, 20, 40, 60, 80, 100, 120 \) and 170 cm (± 0.5 cm). In all cases only vertical drops were considered, and for \( h = 170 \) cm, an extra launch pad was required. We used three different kinds of intruders to obtain repulsive, attractive or neutral interaction with the grains. For the two first cases, the intruder consisted of an acrylic disk of diameter \( D = 3.26 \pm 0.01 \text{ cm} \) with 13 magnets (identical to those of the granular bed) inserted in perforations along the disk circumference. Therefore, the projectile can experience either repulsive or attractive interactions with the granular bed depending if the magnets of the intruder have their dipoles parallel or anti-parallel to the magnetic dipoles of the grains. For the neutral case, an acrylic disk with a lead core was used to obtain the same mass. For the three intruders, the total mass was \( M = 8.2 \pm 0.1 \text{ g} \). Each impact was repeated five times and filmed with a high-speed camera Photron SA3 at 1000 fps. The trajectories of projectiles and grains were obtained using the software ImageJ®.
3. Molecular dynamic simulations

In parallel to the experiments, we developed molecular dynamics simulations (MDS) implementing the Velocity-Verlet algorithm in a Matlab® routine. This scheme can be obtained from Taylor expansions for the positions r(t) and velocities v(t). According to this method, at time \( t + \Delta t \) we have:

\[
\mathbf{r}(t + \Delta t) = \mathbf{r}(t) + \mathbf{v}(t)\Delta t + \mathbf{a}(t)\Delta t^2/2
\]

\[
\mathbf{v}(t + \Delta t) = \mathbf{v}(t) + (\mathbf{a}(t) + \mathbf{a}(t + \Delta t))\Delta t/2
\]

where \( \mathbf{a}(t) \) is the total force acting on the particle divided by the mass. In this case, we consider the weight of one grain \( \mathbf{W} = mg \), the repulsive magnetic force \( \mathbf{F}_m \), and the friction force \( \mathbf{F}_f \) due to the interaction of the magnets with the frontal-rear walls. Here, the magnetic force was calculated by adding all the contributions from the interacting magnetic dipoles [24]. In our configuration, the dipoles point out in the same direction and perpendicular to the frontal wall. Under these conditions, the force of a magnetic dipole \( \mathbf{p}_m \) acting on a second identical magnetic dipole is given by:

\[
\mathbf{F}_m = \frac{3\mu_0\mu_r^2}{4\pi r^3} \mathbf{p}_m
\]

where \( \mathbf{r} \) is the inter-particle radial distance. On the other hand, in Ref. [23], \( \mathbf{F}_f \) was found very important to reproduce the discharge of magnets from a silo because this force increases when the magnets approach each other, generating a torque \( \mathbf{\tau} = \mu_r \times \mathbf{B} \) that tries to turn over the magnets. Since the walls avoid the magnet inversion and \( \mu_r \perp \mathbf{B} \), the friction force can be estimated from the torque acting on each particle, \( \mathbf{\tau} = \mu_r \mathbf{B} = \mu_r \mathbf{F}_f \mathbf{d} \), where \( \mathbf{F}_f \) is the normal force exerted by the particle on the wall at each point of contact and \( \mathbf{d} \) is the particle diameter. Considering a friction coefficient \( \mu_r \), the dynamic friction force on a magnet is \( \mathbf{F}_f = \mu_r \mathbf{F}_n = \mu_r \mu_s \mathbf{B} / \mathbf{d} \). Thus, by adding all the interactions, the acceleration of the particle can be expressed as:

\[
\mathbf{a}(t) = \mathbf{g} + \left( \sum_{n=1}^{N-1} \mathbf{F}_{m}(t) - \frac{\mu_r \mu_s \mathbf{B}(t)}{d} \right) / m
\]

By using the above equations at each step of time to compute the positions and velocities of all the particles, we run our simulation being careful with the parameters associated to the projectile because it is almost 10 times heavier than the rest of the particles, and its magnetic field can be positive, negative or zero, corresponding to a repulsive, attractive and a neutral case, respectively. Our simulations always have two parts, first we drop the bed of particles just as we made experimentally, and then, we drop the projectile on the repulsive bed at equilibrium from a height \( h \). From the simulations we follow the projectile position and the kinetic energy of the particles at each time step until achieving the relaxation of the system. The numerical algorithm was validated by

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**Fig. 2.** Time lapse snapshots taken from videos of experiments for impacts with a) repulsive interaction, b) attractive interaction, and c) neutral interaction. The penetration is divided in different stages observed in each case. The images on the right indicate the approximate size of the interaction zone for each case and the paths of the affected grains.

**Fig. 3.** a) Images showing the displacement of the grains for each kind of interaction. The darkest part of each path is the final position of the grain. b) Local density of the granular bed just above and below the intruder when the maximum penetration is reached in the three cases. Rectangular regions were selected following a similar analysis performed in Ref. [25].
comparing the simulations with the experiments, and then, the simulations were used to extend the range of impact velocity, as well as the magnetization of the particles.

4. Results

4.1. Experimental results:

Let us first describe the phenomenology. Fig. 2a–c show snapshots of the impact and penetration for repulsive, attractive and neutral interactions, respectively. For the repulsive case (Fig. 2a), the intruder compresses the bed and forces the grains to get closer to each other accumulating magnetic potential energy, like a spring. When the maximum depth $z_{\text{max}}$ is reached, the restoring force due to the bed produces the rebound of the projectile. Some minor subsequent rebounds can be observed before the projectile stops at a final depth $z_f < z_{\text{max}}$. For the attractive case (Fig. 2b), the intruder is first accelerated when it approaches the bed due to the magnetic attraction of the grains; then, a layer of magnets is rapidly attached to the perimeter of the intruder. Due to the sudden increase of the intruder mass ($\approx 5.6$ g), the momentum conservation in an inelastic collision implies a rapid deceleration from the impact speed $v_i$ to $v = M v_i / (M + 14 m) \approx 0.6 v_i$. The deceleration is amplified by the increase of the effective size of the projectile; nevertheless, the intruder goes deeper than in the repulsive case due to its increase in weight ($\sim 1.73$ times larger). Moreover, the repulsive interaction of the attached grains is partially compensated (shielded) by the attractive interaction of the magnets inside the disk, and then the projectile interacts with a smaller number of grains and can penetrate deeper into the bed. For the neutral intruder (Fig. 2c), the total absence of magnetic interaction with the grains generates a lower compression of the magnetic bed, which allows the deepest penetration of the three cases. In all scenarios, the projectile stops when its weight is balanced by the magnetic interactions and the friction forces due to confinement. For videos of the experiments described above, please see Supplementary Movie 1.

Fig. 2 also shows the interaction zone of each intruder and the paths followed by the interacting grains. Note that the compression zone is considerably larger for the repulsive projectile ($\sim 5D$) than for the other two cases ($\sim 3D$). Fig. 3a shows that for the repulsive case the grains move away from the penetration trajectory drawn by the intruder, whereas for the attractive and neutral cases the grains flow to the area above the projectile. These flows lead to a difference in the local density $\sigma$ of the granular bed above and below the projectile, see Fig. 3b. The marked difference of such densities for the repulsive case allows the projectile rebound because a larger concentration of grains in the lower zone implies a larger magnetic restoring force, and the grains in the upper zone are not enough to mitigate the rebound. For the attractive and neutral cases, this difference in densities is less important and the rebound is suppressed or negligible.

Fig. 4 shows the intruder penetration depth $z$ and its velocity $v$ as a function of time, $t$, for the three types of intruders impacting from different initial heights $h$. Each line represents the average of five trajectories and the velocity was obtained from the first derivative of $z$. Note that the kinematics described above is reproduced for the different impact velocities: i) a maximum depth followed by oscillations for the repulsive object due to the bed compaction and elastic response, ii) the larger deceleration for the attractive projectile (more pronounced slopes) which stops at a shorter time ($0.06$–$0.08$ s), and iii) a larger final depth for the neutral intruder that has less interaction with the grains. In all cases, $z(t = 0) = 0$ corresponds to the bed surface and $v_i = v(t = 0)$ to the impact velocity. The positive values of $v(t)$ only observed for the repulsive projectiles correspond to the rebound when the projectile moves upwards. The oscillatory dynamics is similar to the one observed during the penetration of an object into a viscoelastic miscellar fluid [9]. For the attractive case, the larger values of $v(t = 0)$ reflect the magnetic attraction of the bed that slightly increases the projectile velocity before impact.

From the above dynamics, we focus on three parameters and their dependence on the impact velocity: the total penetration time $t_p$, the final depth $z_f$ for the three cases, and the maximum depth $z_{\text{max}}$ before the first rebound for the repulsive case. Fig. 5a shows that $t_p \approx 0.1$ s for the neutral and attractive cases regardless of $v_i$. A practically constant penetration time independent of the impact velocity has also been reported for impacts into frictional granular materials [16]. The repulsive
reach a constant value, see inset in Fig. 5c. The last two results are similar to those found for impacts in viscoelastic fluids [9]. Nevertheless, these observations are valid in a short range of impact velocities and larger values cannot be explored experimentally due to the set-up limitations. For this reason, we developed numerical simulations in order to explore the dynamics in a wider range.

4.2. Numerical results

Fig. 6 shows the results obtained from MD simulations for projectiles released from \( h = 0 \) to 800 cm above the magnetic granular bed. The kinetic friction coefficient \( \mu_k \) in Eq. (2) was estimated numerically for each kind of intruder in order to obtain, on average, the best fit for the experimental curves. For the repulsive projectile \( \mu_k = 0.42 \), for the attractive projectile \( \mu_k = 0.36 \) and for the neutral case \( \mu_k = 0.28 \). The magnetic moment \( \mu_m \) was considered equal to the experimental value \( \mu_m = 55 \text{ mA/m}^2 \). After fixing these parameters, simulations were carried out to obtain the dynamics for different impact heights. Remarkably, the trajectories shown in Fig. 6a reproduce the experimental observations for the corresponding kind of interaction. The numerical approach allowed us to include results for \( h = 5 \) cm and \( h > 170 \) cm.

Fig. 6 b shows that the simulations also reproduce the experimental results concerning the penetration time, with \( \tau \) largely independent of the impact velocity. The only difference is that the simulations do not reproduce the peak in \( \tau \) for \( h \approx 20 \) cm observed experimentally for the repulsive case. The penetration depth \( z_p \) as a function of \( h \) is also in good agreement with the experiments in the range \( h = 0 \) to 170 cm (Fig. 6c); however, instead of a power-law scaling, \( z_p \) is better described in a wider range of \( h \) by a logarithmic dependence of the form \( z_p = z_p^\text{max} + b \log(h) \), where \( b \) and \( z_p^\text{max} \) are fitting parameters. In fact, the power scaling proposed for the experiments (green dashed line in Fig. 6c) fails for large values of \( h \). The simulations also confirmed that the rebound height \( h_r \) reaches a constant value (Fig. 6d), but in this case \( h_r \approx 3.3 \pm 0.2 \) cm, larger than the value found experimentally.

The numerical simulations were also used to explore the effect of varying the magnetic field of the particles in the projectile dynamics. This is clearly an advantage considering that changing systematically the magnetic dipolar moment of the magnets is difficult from the experimental point of view. Fig. 7a shows examples of repulsive granular beds built numerically for different values of magnetic strength \( M \). The value of \( M \) was normalized with \( M_{\text{max}} \), which corresponds to the experimental value \( \mu_m = 55 \text{ mA/m}^2 \). Thus \( M/M_{\text{max}} = 1 \) corresponds to a numerical granular bed that is equivalent to the experimental bed. When \( M \) is reduced, the particles get closer and the density of the bed increases considerably. For \( M/M_{\text{max}} = 0.01 \), most of the grains are practically in contact because the magnetic repulsion becomes negligible. Fig. 7b shows \( z \) as \( \tau \) for the three intruder-grains interactions for an intruder released from \( h = 100 \) cm. When the magnetization is reduced, the projectile stops at a shallower depth in all cases. This can be understood considering that the density of the bed increases and the projectile must replace a larger amount of grains during its penetration. When the magnetization is negligible, the three interactions become basically neutral and the final depth is approximately the same for the three intruders.

It is important to mention that the density of the bed is not homogeneous and has a Janssen-like increase with \( z \) [23]. Fig. 8a shows the volume fraction \( \phi \) of the bed as a function of \( z \) obtained from the four granular beds depicted in Fig. 7a. Note that the volume fraction of the bed used in the experiments (black line) is well approximated by the numerical result (red line), with \( \phi \) increasing from 0 to \( \approx 0.2 \) at the bottom of the bed (see experimental and numerical cases for \( M/M_{\text{max}} = 1 \)). The maximum packing fraction is obtained for \( M/M_{\text{max}} < 0.1 \), with \( \phi \approx 0.82 \), which reflects the fact that the magnetic disks form almost a hexagonal array at the bottom of the bed. The hexagonal close packing for a two-dimensional array of disk is \( \phi_{\text{hcp}} = \pi/\sqrt{12} \) [26], but the confinement effects and the residual magnetic field avoid the perfect crystallization
Fig. 6. Numerical results: a) \( z \) vs \( t \) obtained from simulations (MDS) for the three types of interactions. b) Penetration time \( \tau \) vs \( h \) for the three different intruders from MDS compared with the experimental data (Exp). c) Final depth \( z_F \) as a function of \( h \) for the three intruders. Solid lines correspond to a logarithmic dependence of the form \( z_F = z_0 + b \log_{10}(h) \). The dashed line corresponding to \( z_F \propto h^{2/3} \) (\( \propto v_0^{1/3} \)) only fits the data in a short range. d) \( h_r \) vs \( h \) obtained from numerical simulations (*) and compared with experiments (•).

Fig. 7. Effects of magnetic strength: a) Granular beds obtained from simulations (MDS) using magnets with different magnetic strength \( M \) normalized with \( M_{\text{max}} \) equal to the experimental value. b) Penetration dynamics obtained from numerical simulations of repulsive, attractive and neutral intruders penetrating beds of grains with different values of \( M \).

in our system. Fig. 8b shows that \( z_F \) decreases as the magnetization is reduced for the three intruders, which is due to the increase of packing fraction. Additionally, Fig. 8c shows that the magnitude of the rebound height \( h_r \) for the repulsive case has a maximum value for \( M/M_{\text{max}} \sim 0.2 \) and then decreases slightly for larger magnetizations.

Finally, the numerical simulations were used to explore the effect of friction due to the confinement on the projectile dynamics. The friction term was neglected in the numerical simulations by making \( \mu_k = 0.01 \) for the frontal and rear walls. Fig. 9a shows snapshots taken from the simulations for the three intruders and the corresponding trajectories for \( h = 20 \text{ cm} \). We confirmed that for the three cases the projectile sinks into the fluidized bed until reaching the bottom of the container, whereas the frictionless grains remain in motion because the dissipation of the kinetic energy provided by the impact is negligible. One can distinguish a slight oscillation in the repulsive trajectory in Fig. 9b at \( t \sim 0.2 \text{ s} \), similar to the oscillations observed during the transient motion of a sphere.
falling into a highly elastic aqueous polyacrylamide solution [13]. The rebounds at $h = 33$ cm are due to the impact of the projectile with the container bottom. Movies of the simulations showing the impact of the three types of projectiles considering and neglecting friction can be seen in Supplementary Movies 2 and 3, respectively. These numerical results are in clear contrast with the experimental results, where the friction with the lateral walls cannot be avoided due to the torques produced by the magnetic field acting on the magnetic dipoles of the grains. This friction generates the fast dissipation of the projectile kinetic energy. Fig. 9c shows the temporal evolution of the kinetic energy of the projectile and the granular bed normalized with the kinetic energy of the projectile at impact ($KE_0$), obtained using particle tracking from videos of experiments with the three types of intruders released from $h = 20$ cm. At impact, $KE/KE_0 = 1$ for the projectiles and 0 for the grains. Then, the projectiles start to loose energy that is transferred by collisions to the grains. However, the cumulative kinetic energy on the bed never reaches $KE_0$ because an important fraction of energy is dissipated by friction with the walls. In fact, at any time $t$, the sum of the kinetic energy of the grains is always less than 20% of the energy at impact, which indicates the fast dissipation of energy with the wall. Eventually, all the kinetic energy is dissipated and the intruder and the particles reach the repose. These experimental results reveal that, even though collisions between particles are suppressed, the system is considerably dissipative due to the lateral confinement.
5. Model

The previous results reveal several similarities between the dynamics reported here and the penetration of a projectile into a viscoelastic micellar fluid reported in Ref. [9]. Moreover, the fact that the velocity of the repulsive projectile describes oscillations indicates that a good candidate for the drag force is the Maxwell model discussed in the introduction. Therefore, the dynamics of the projectile considering the magnetic bed response on impact can be described with the differential equation of motion of a damped harmonic oscillator:

\[ m \ddot{z} + m g_0 - k z - \beta \dot{z} \]

The equation can be rewritten in the form:

\[ \ddot{z} + 2 \lambda t + \omega^2 z = g_0, \]

where \( \omega = \sqrt{g/m} \) is the undamped angular frequency of the oscillator and \( \lambda = \beta / 2m \). Here, \( g_0 \) is a modified acceleration of gravity that includes the repulsive or attractive effect of the granular bed on the falling projectile. The system is underdamped if \( \lambda^2 - \omega^2 < 0 \). In such situation, the projectile would oscillate within the fluid with decreasing amplitude over time with the solution of Eq. (3) given by:

\[ z(t) = -\frac{g_0}{\omega^2} + A \exp(-\lambda t) \sinh \left( \frac{t \sqrt{\omega^2 - \lambda^2}}{2} + \phi \right) \]

On the other hand, the system is overdamped if \( \lambda^2 - \omega^2 > 0 \). With such condition, the projectile is stopped without oscillations following the equation:

\[ z(t) = -\frac{g_0}{\omega^2} + A \exp(-\lambda t) \sin \left( \frac{t \sqrt{\lambda^2 - \omega^2}}{2} + \phi \right) \]

The constants \( A \) and \( \phi \) can be found analytically using the known initial conditions \( z(t = 0) = 0 \), \( z(t = 0) = v_0 \), and the final position of equilibrium \( z(t = \infty) = z_F \). The elastic and viscous constants \( \lambda \) and \( \omega \) are free parameters.

The experimental results were fitted with the above model using Mathematica® and also compared with the MD numerical simulations in Fig. 10. For clarity, only data for two impact heights \( h = 20 \text{ cm} \) and \( h = 120 \text{ cm} \) are shown. For the repulsive and attractive cases we used Eq. (4), and for the neutral case the Eq. (5). Remarkably, the main dynamical features found in the experiments and numerical simulations (damped oscillations and overdamped trajectories) are well reproduced by the model. By fitting the experimental data, we found that the elastic and viscous constants for the attractive case must be considerably larger than the corresponding values for repulsive and neutral cases, reflecting the fact that the effective size of the projectile increases when the magnets attach the projectile surface when it reaches the magnetic bed.

6. Discussion

The impact of a solid object in Newtonian fluids [27] or in granular materials [4] is typically analyzed in terms of the Froude number, \( Fr = v_p^2/d \cdot g \), where \( g \) is the gravitational acceleration, \( v_p \) is the projectile velocity and \( d \) its diameter. For impacts into viscoelastic fluids, it is necessary to take into account elasticity effects [9]. By introducing the elastic Froude number as the ratio of kinetic energy to elasticity, \( Fe = \Delta \rho v_p^2/G \) (where \( \Delta \rho \) is the difference between the projectile and target densities and \( G \) is the elastic modulus of the fluid), it was shown in Ref. [9] that the maximum depth before rebound for low impact speeds is proportional to \( Fe^{1/3} \), i.e. \( \propto \sqrt[3]{\rho v_p^2} \). In the case of a viscoelastic fluid, the polymers or micellar tubes responsible for the elasticity are stretched as the projectile sinks, generating a restoring force that makes the projectile bounce [9,13]. In our system, the elastic response of the bed when it is compressed by the intruder is due to the repulsive nature of the magnetic force that increases as the radial distance \( r \) between magnets decreases. Therefore, the power-law scaling found in Fig. 5c could be associated to the elastic response of the magnetic granular bed.

On the other hand, for higher impact speeds, the logarithmic saturation of \( z_r \) vs \( h \) found in the numerical results in Fig. 6c indicates that the medium becomes harder to penetration as the deformation rate increases. This rheological behavior resembles the shear thickening response to deformation of a non-Newtonian fluid [14]. When the compression produced by the impact is slow, the matrix of grains has enough time to be rearranged, and the grains can reach zones of lower repulsive potential. This rearrangement is not allowed when the compression is fast; thus, the matrix of magnets becomes locally arrested because the
torque generated on each magnet increases the friction force against the walls. Additionally, the increase in density at larger depths quantified in Fig. 8a also contributes to the resistance to penetration.

Concerning the rebound height $h_r$, one could expect that its magnitude should increase with $v_0$ since more magnets are compressed as the projectile sinks. However, the saturation of $x_p$ also implies that $h_r$ must reach a constant value. Moreover, the magnetic grains above the intruder act as a damping mechanism of the rebound. In the case of micellar fluids [9], the constant value for $h_r$ was associated to confinement effects. Since confinement is of great relevance in our experiments to avoid the collapse of the magnetic bed, this could also have an important effect. Indeed, in a typical granular material, the kinetic energy at impact is dissipated by friction inside the bed. Here, there is no direct contact between the grains and the projectile, and the energy dissipation is associated to friction between the grains and the confining walls. Without friction or confinement, the projectile would sink indefinitely through the magnetic bed without rebound, as it was shown numerically in Fig. 9.

The repulsive and attractive dynamics observed in our study resemble the surprisingly complex dynamics of different scenarios. For instance, small charged dust particles in space environments experience electrostatic long range interaction that produces levitation, agglomerations, repulsion, and capture of particles [28]. The attachment of magnets to the surface of attractive projectiles resembles the particle trapping process induced by cohesive forces when air bubbles move in a liquid with suspended micro-particles [29]. Similarly, air bubbles generated by a jet of grains entering water are decelerated due to the attachment of grains to the bubbles surfaces [30]. Such deceleration is also observed here in the dynamics of the attractive projectile due to the attachment of magnets to its surface.

7. Conclusions

The impact and penetration of a projectile into a repulsive magnetic bed was studied experimentally and using molecular dynamics simulations. For the first time, a long range interaction between the intruder and the granular bed was introduced and systematically analyzed. In a consolidated granular material, where inter-particle friction and collisions are the main mechanisms of energy dissipation, the maximum penetration of a projectile $x_p$ scales as the $1/3$ power of the energy of the intruder at impact: $x_p \propto V_0^{1/3}$ [16]. In our system, there is no friction or collisions among particles, and the only mechanism of energy dissipation is friction with the confining walls. Although the $1/3$ power-law scaling holds in the short range of parameters explored in our experiments, the simulations revealed a logarithmic saturation of $x_p$ vs $h$. The simulations also showed that the projectile sinks to the bottom of the granular bed when the friction term is neglected, like in a liquid. As the magnetization of the grains is reduced, the bed packing fraction increases and the dynamics of repulsive, attractive and neutral projectiles becomes more similar. Due to the repulsive nature of the magnetic interaction that scales with the interparticle separation $r$ as $r^{-4}$, the potential repulsive energy of the magnets increases when the granular bed is compressed by the impact, generating a restoring force that makes the projectile bounce. The dynamics is similar to the bouncing dynamics of a projectile penetrating into a viscoelastic micellar fluid [9,13], and it was described using a Maxwell model [10] that is able to capture the main features of the projectile dynamics.

Magnetic particles have been previously used to mimic or study the dynamics of several systems; for instance, the discharge of a container with repelling grains flowing through an orifice [22,23] resembles the flow of cars passing through a bottleneck. Dense structures of millimetric magnetic beads show a response to deformation similar to solids in condensed matter physics depending on the crystalline structure of the array [31]. Here, we showed the viscoelastic-like behaviour of a bed composed of magnetic repelling grains on impact. We believe that our study can inspire further research on granular materials using long-distance interacting particles (the dynamics of avalanches or granular gases composed of repelling particles are promising). Moreover, magnetic repelling grains can be used for developing a new kind of magnetic granular damper [32] avoiding the wear of the particles that is produced in classical granular dampers [33,34].

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Supplementary material


References


